Asset Pricing Dynamics under Expectations Diffusion: the Effect of Social Influence, Homophily and Stubborn Agents

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Abstract

[Very preliminary version. Comments welcome]. We study the feedback between expectation diffusion and asset price dynamics in a simple asset market model in which local interaction among traders and agents' heterogeneity play a crucial role. Investors are connected through a social network and in each period agents communicate locally with one of the individuals belonging to their neighbourhood. They then may switch between different rules, which govern their expectations about future stock prices, taking in consideration their own performance relative to that observed at agents in their neighbourhood. Hence, the rate of switching to a different expectation rule does not only depend on the relative performance of the rules but also increases with the degree of (local) belief heterogeneity in the population and the second effect has a crucial impact on the characteristics of the emerging dynamics.

JEL classification: C63; D84; D85; G12; G41.

Keywords: Bubbles; Social interactions; Homophily; Stubborn agents; Heterogeneous expectations.

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1 Introduction

Word of mouth and communications in financial markets have shown to be important features to understand price dynamics and replicate stylized facts. Therefore, social inteactions and communication should be modelized to achieve a better understanding of financial systems and the literature on opinion evolution applied to social networks becomes, then, relevant. One can spread between Bayesian and Non- Bayesian updating mechanism whether each agent has knowledge of the general model of the economy. If the answer is negative, agents use simple local updating strategies. DeGroot's model is a standard non-Bayesian model in which individuals update their beliefs as a weighted average of their neighbours beliefs, relating, thus, to averaging consensus and gossip algorithms, e,g, DeMarzo, Vayanos, and Zwiebel [8] and Golub and Jackson [9]. Disagreement, clustering and polarization represent important feature to investigate in this framework. A lot of work has been done regarding the asymptotic behaviour of opinions, e.g. Axelrod [4], Krause [11],Li et al. [12] with no enlightening with respect to opinion fluctuations and the role of stubborn agents in the expectation diffusion process. Acemoglu, Ozdaglar, and ParandehGheibi [1] and Acemoğlu et al. [2] consider the interaction of forceful agents, which always stick to the initial opinion, and herding agents, which, instead, are affected by others' beliefs.

All the former non-Bayesian approaches involved a certain type of imitation without giving importance to the relative performances of agents' beliefs. The switching process, e.g. Brock and Hommes [5, 6] represent a relatively simple mechanism which enables to change strategy over the relative performance measure. The mechanism still include imitation, but add to it information about the fitness of different strategies. Individuals should change their opinion towards the alternative that shows the higher performance in the previous period. Panchenko, Gerasymchuk, and Pavlov [13] investigate the effects of networks structures on asset price dynamics, allowing information diffusion through local communication. They found that different network structures, namely fully connected, regular lattice, small world and random graph, affect information transmission creating persistent inefficiencies which induces grater instabilities and higher deviations in the price dynamics.

Based on extensive simulation analysis, we also study how key properties of the dynamics of asset prices and beliefs, in particular the emergence of fluctuations, is influenced by the topology of the social network. More specifically, we show that the degree of clustering in the network, induced by homophily of the network along some fixed characteristic of the agents (e.g. geographic location, educational background), has important implications for the observed dynamics. Furthermore, the existence of stubborn agents, who are not willing to change their expectation rules, fosters the emergence of fluctuations adding another level of heterogeneity to the system.

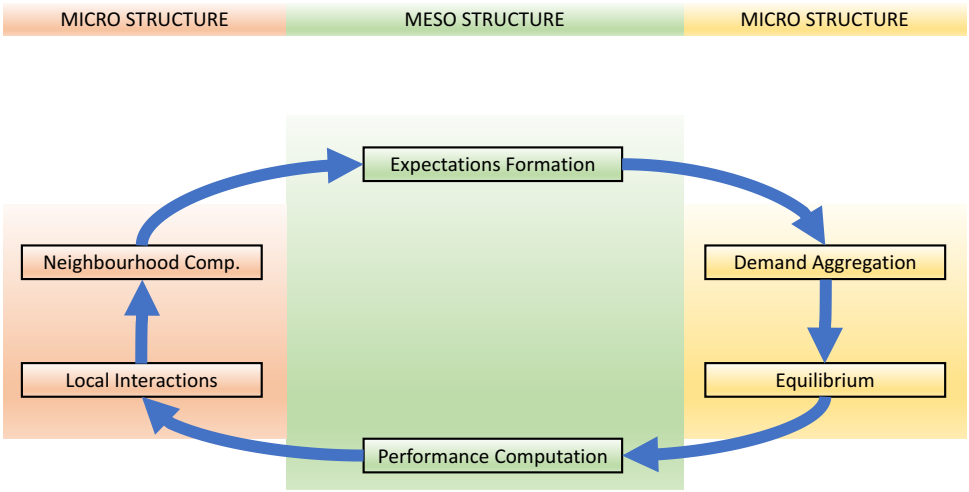


Figure 2.1: Model Algorithm

2 Model Framework

We divide the model framework into three sections: micro structure, meso structure and macro structure. The micro structure intends to provide a description of the agents in the market, focusing on their characteristics, their beliefs and their typology. The meso structure, instead, captures the expectations dynamics where both local interaction and performances play a crucial role. Finally, the macro structure presents market aggregation and clearing process.

In figure 2.1 we try to summarize the model's algorithm: after the beliefs on future price are formed, the market demand is aggregated and, then, trades occur, and, through the walrasian auctioneer market clears. At the end of the period, the average performance for each belief is computed and, with a certain probability, it is compared to the neighbour's. At this stage, the number of agents belonging to a certain type is updated and new expectations are formed.

2.1 Micro Structure

We consider a finite set $N = \{1, \ldots, n\}$ of interacting agents which can be thought as nodes of an undirected graph $G(N, E)$, where E represents the set of edges between each node. Links between node are given by the entries, $[0, 1]$, of the adjacency matrix $A_s(n \times n)$ which varies among different network topologies and simplifies the construction and the analysis of the framework. The interpretation is that $A_{i,j} = A_{j,i} = 1$ indicates that agent i and j are linked, and the symmetry restricts attention to undirected networks.

Agents' vector is associated with a randomly generated and time invariant vector of characteristics, $Ch(1 \times n)$, representing any general cultural feature related to human value

environment, and which entries are $[0, 1, \ldots, h]$. On these characteristics, the concept of homophily[1] is developed.

Moreover, everyone holds a starting belief which can change in time due to social interaction and relative performances. The entries $[0, 1, \ldots, bf]$, representing the set of bf heuristic rules available in the market, of the beliefs vector considered, $Bf(1 \times n)$, are uniformly distributed and, hence, each expectation has the same starting fraction of the population.

We distinguish between two types of agents: stubborn and herding. The former type, generally identified by few agents, never consider the information receive by others. It can be though as formed by experts or opinion leaders which wish to influence the rest of the community, or, from an opposite perspective, simply biased individual favouring one opinion. Herding agents, instead, represent the majority of the population and may update their beliefs according to a switching mechanism that will be discussed in the next section. Thus, one should notice the disproportional information exchange in a meeting between herding and stubborn individuals. The distribution of agents' type across beliefs is uniform and their relative percentage represents a critical variable of the model. In particular agents' type is related to the characteristic vector Ch: each characteristic is associated with a certain(varying) percentage of biased individuals having a specific belief.

2.2 Meso Structure

Agents are connected through a social network and communicate locally with one of the individuals belonging to their neighbourhood exchanging informations. This meeting mechanism captures information regarding beliefs performances and neighbourhood composition in terms of adoption of a certain belief. In fact, they may switch between different rules taking in consideration their own performances relative to that observed at agents in their neighbourhood. Hence, the rate of switching to a different expectation rule does not only depend on the relative performance of the rules but also increases with the degree of local belief heterogeneity in the population. It is given by:

$$P_t(i \quad switches \quad to \quad bf = k) = \frac{N^i_{k,t-1}}{\sum_{bf} N^i_{bf,t-1}} \frac{e^{(\beta U_{k,t-1})}}{\sum_{bf} e^{(\beta U_{bf,t-1})}} \tag{1}$$

The first component on the right hand side expresses the local heterogeneity in beliefs, while the second component is created upon the discrete choice multinomial logit model.

N^i_{bf} is the number of i' neighbours with a certain belief bf and U_{bf} is the performance measure of a certain belief. We can distinguish in the fitness measure between a deterministic element reflecting current performances, and a stochastic element showing

[1]Lazarsfeld et al. (1954) define homophily as the tendency of individuals and their associates to have similar beliefs, ethnicity, age, religion, party affiliation, education, etc. McPherson et al. (2001) provide a review of this literature. For a more recent research on homophily, see Easley and Kleinberg (2010).

measurement error of agents, or behavioural biases, time and agents independent. Moreover, we assume that the stochastic element of the performance is observed by everyone and follows a Gumbel (or Boltzmann-Gibbs) distribution, then the probability of selecting a certain rule depends on relative performances.. Finally, β is defined as the intensity of choice affecting the impact of the random component on the fitness function and it measures the relative importance of performance comparison over neighbourhood composition. Therefore, higher intensity of choice produces two effects: on one hand, it makes agents more sensitive to differences in performances, increasing their willingness to switch belief; on the other hand, it leads to lower heterogeneity in the population, decreasing the meeting probability and the switching rate.

Immediately it comes out that agent's probability of changing strategy depends on the probability of meeting someone, within his neighbourhood, with different belief. Thus, if at a certain period, one is surrounded only by homogeneous agents, he will not have the chance to compare the strategies and therefore he does not receive any informational content from the meeting.

2.3 Macro Structure

We consider agents interacting in a financial market with two assets like in Brock and Hommes [6]. The reserve asset is an elastically supplied risk free bond which pays constant gross return $R = 1 + r$. The risky asset, i.e. stock's shares, gives uncertain dividend y_t and its price p_t pops up from the trading activity on the market at each period. Let $S_{i,t}$ and $B_{i,t}$ denote respectively the number of shares and of risk free asset hold by investor i at time t. The risk aversion coefficient is common to all investors which are myopic mean-variance maximizers. Thus, the wealth dynamics comes from

$$W_{i,t+1} = S_{i,t}(p_{t+1} + y_{t+1}) + B_{i,t}(1 + r) \tag{2}$$

$$W_{i,t} = S_{i,t}p_t + B_{i,t} \tag{3}$$

Therefore

$$W_{i,t+1} = R(W_{i,t} - p_t S_{i,t}) + (p_{t+1} + y_{t+1})S_{i,t} \tag{4}$$

The mean variance optimization is

$$\max_{S_{i,t}} \left\{ E_{i,t}[W_{i,t+1}] - \frac{a}{2} V_{i,t}[W_{i,t+1}] \right\} \tag{5}$$

$$\max_{S_{i,t}} \left\{ S_{i,t} E_{i,t}[p_{t+1} + y_{t+1} - Rp_t] - \frac{a}{2} S_{i,t}^2 V_{i,t}[p_{t+1} + y_{t+1}] \right\} \tag{6}$$

Solving the optimization problem, the first order condition leads to the demand function of the single investor

$$S_{i,t}(p_t) = \frac{E_{i,t}[p_{t+1} + y_{t+1}] - Rp_t}{aV_{i,t}[p_{t+1} + y_{t+1}]} = \frac{E_{i,t}[p_{t+1} + y_{t+1}] - Rp_t}{a\sigma^2} \tag{7}$$

where we have assumed constant and homogeneous belief of all investors over the conditional variance of excess returns, and equals to σ^2.

Each demand/supply function is inversely related to investors' risk aversion which determines their position on the market. If $a \ll 1$ a low level of uncertainty is already enough to strongly decrease asset attractiveness, and vice versa if $a \gg 1$. Moreover we would like to stress that the demand function comes from a Constant Absolute Risk Aversion (CARA) utility function and does not include the wealth of the trader. Indeed, the amount of asset purchased by the investor is independent from his current and past wealth, instead, it is dramatically affected by its expectation on future excess return.

Market demand will depend on investors fractions n_t^{bf}. At every period the market is in walrasian equilibrium with demand equal to supply. Since the demand/supply function of investors is strictly decreasing, there exist a unique clearing price:

$$p_t = \frac{\sum_{bf} n_t^{bf} E_t^{bf}[p_{t+1} + y_{t+1}]}{R} \tag{8}$$

where we have assumed zero outside supply of stock' shares.

Moreover assuming that traders have common correct expectations about the exogenous dividend process, allows us to treat the models in deviations from a benchmark and implies that there is no uncertainty about fundamentals in the model. The heterogeneous expectations of investors is the reason behind doubts on future variation of asset price.

In order to have rational benchmark, we consider a world where all agents are equals and with homogeneous constant expectations on future prices. Therefore traders forecast zero mean and zero variance for asset returns. The market clearing equations becomes

$$p^* = \frac{\overline{y}}{R-1} \tag{9}$$

which is thought as the fundamental price [2]. The discounted sum of all future dividends is known as the Gordon model and it is constant under rational expectations. It corresponds to the fixed point and it is not affected by any behavioural consideration of the traders.

At each trading moment, every trader chooses a forecasting rule. All price expectations are of the form:

$$E_t^{bf}(p_{t+1} + y_{t+1}) = [p^* + \overline{y}] + [b_{bf} + g_{bf}(p_t - p^*)] \tag{10}$$

[2]It is the price that would prevail in efficient markets populated only by rational agents. It is an arbitrage market relation.

where the first component on the right hand side represents the rational expectation on future price and dividend while the second gives the idea of deviations through a simple general heuristic based on a constant bias parameter b_{bf}, a constant trend parameter g_{bf} that amplify any deviation from the fundamental price.

The selection of the forecasting rule is also based upon a performance measure composed by a common observed deterministic component reflecting current performances, i.e. the average of the individual realized excess profits, and an unobserved stochastic component showing measurement error of agents, or behavioural biases, independent over time and over agents

$$U_t^{bf} = \left[(p_t + y_t + Rp_{t-1})S_{t-1}^{bf}\right] + \left[\frac{1}{\beta}\varepsilon_t^{bf}\right] - C_{bf} \tag{11}$$

where β is the *intensity of choice* affecting the impact of the random component on the utility, while the parameter C_{bf} represents an average of cost for each period needed to have a relatively "sophisticated" predictor.

3 Computational Analysis

We conduct extensive simulation analysis in order to underline how key properties of the dynamics of asset prices and beliefs, in particular the emergence of fluctuations, is influenced by the topology of the social network. The numerical analysis is run over a population of 500 agents for 2600 periods.

Although the predictor includes several types of investment strategy, we will take into account only two, *fundamentalist* and *trend follower*. Both strategies have bias parameter $b_{bf} = 0$, while they differ on the trend parameter value g_{bf}. Fundamentalists think that the market price will be at the same level of the fundamental price p^*, or, in other words, that the deviation x from the fundamental will be zero. Therefore they adopt $g_f = 0$. The second forecasting rule believes in trends, positive or negative, and try to exploit them considering $g_{tc} > 1$.

The characteristics' vector considered is a vector with binary entries representing two general features related, for example, to education or geographical location. Therefore we can split the population into two groups, which differ on the belief agents are stubborn on. In other words, a certain percentage of agents with (non variable) characteristic $Ch = 0$ is stubborn on the *fundamental* belief, while investors with characteristic $Ch = 1$ are stubborn on the *trend following* strategy at the same percentage as for the previous group. The total number of stubborn individuals in the market is a key variable of the model.

Then, four different network topologies are investigated: fully connected, random graph, block model and a barbell shape network. The structures selected reflect an ex-

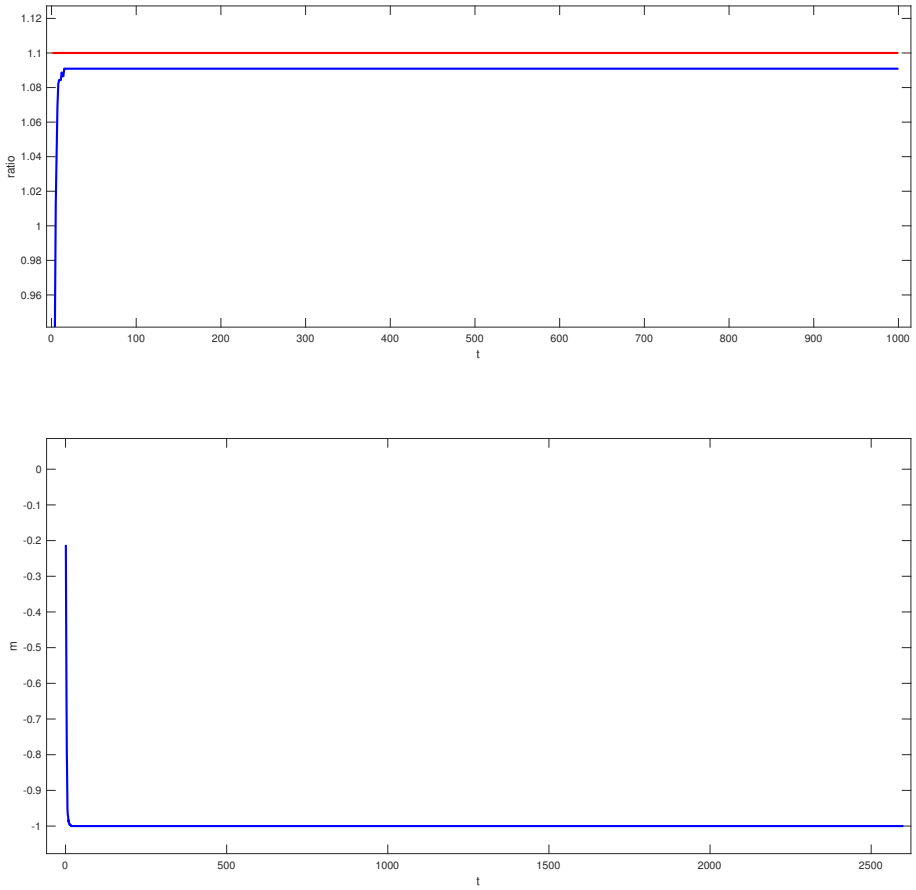


Figure 3.1: 0% level of stubborn agents. Top panel is the Incremental ratio of price differences in blue and constant value of R in red; bottom panel fractions dynamics.

ogenously given homophily level, from zero of the first two, to the maximum of one of the block model, passing by an intermediate rate in the barbell.

0% Stubborn Agents

When no stubborn agents are present in the population, expectations quickly converge to a scenario where trend chasers dominate the market and drive out fundamentalists, and, thus, the price is growing at an almost constant rate sustained by rising fraction of irrational agents. Therefore, no fluctuations emerge from expectations or prices. This result is straight forward: fundamental strategy, for 70 periods on average, is doing positive gross profits not sufficient to overcome the cost for the adoption of their sophisticated strategy. Thus, chartist strategy is always preferred in this time frame until the extreme consequence of driving out the opposite trading rule after 15 periods, on average. Increasing the intensity of choice β has the only effect of a faster supremacy of trend chasers in the market (4 periods on average). See figure 3.1.

In what follows, a brief description of each topology and their simulations are provided.

3.1 Fully Connected Network

The name of the topology under analysis already gives us the insight that all nodes should be linked to each other avoiding self loop. Technically the adjacency matrix A_{fc} has entries equal to zero only on the diagonal, while all the others are equal to one. Eventually, stubborn agents of both groups interact while herding agents guarantee expectations fluctuations. The homophily level in this set up is equal to zero due to the even number of links between the two characteristics groups. Thus, society coincides with a single neighbourhood where potentially information regarding beliefs are always available to everyone at each period of time.

From 1% to 5% Stubborn Agents

Fast fluctuations in prices and expectations emerge, introducing small percentage of stubborn agents, both fundamentalists and chartists. The price goes up pushed by over investing of trend chasers which dominate the market, then, because of negative excess return, they suffer losses while fundamentalists receives positive profits becoming the majority until the price reaches again the fundamental value. There, after few periods, chartists lead the market due to positive cost for the rational strategy, and a new bubble is triggered. It should be stressed how relatively slow is the bubble creation compare to its burst which suddenly occurs.

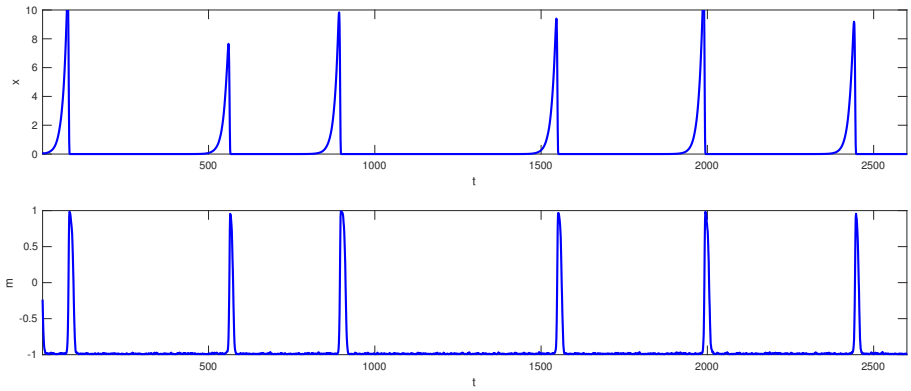
An increase in the fraction of stubborn agents, until the threshold of 0.05 included, affects price fluctuations and the number of shares traded. Price dynamics becomes irregular while the amplitude of its oscillations shrinks from eight to less than four. The shares traded decrease, while expectations swings continue to be wild. See figure 3.2.

The price dynamics behaviour triggered by increasing the intensity of choice from 1 to 100 is dual: at first it forces price non linear fluctuations, while further rises make oscillations regular, less frequent, and wider in amplitude. Fractions fluctuations are wild and regular, swinging from long and constant trend chaser majority to short fundamentalist ruling periods during and after the bubble. In the same spirit, the trade volume starts again and become fleshy. Overall, the cycle dynamics is faster, and both boom and burst happen in a shorter time frame. See figure 3.3.

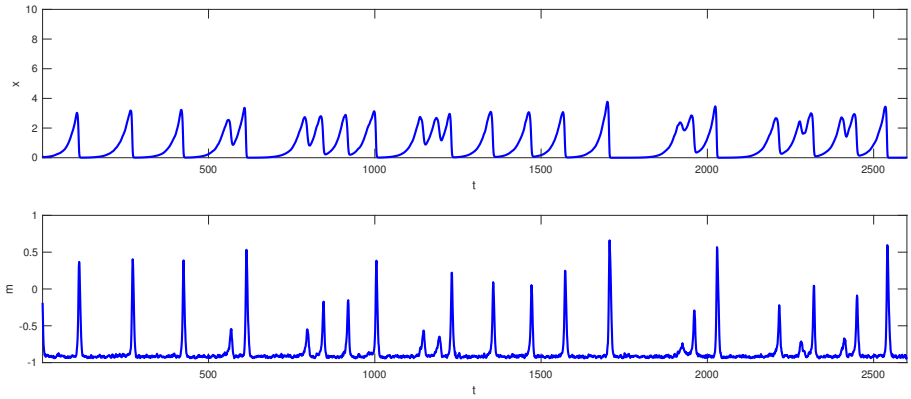
Over 6% Stubborn Agents

Further rise of opinion leaders strongly supports the former changes in price and shares traded: oscillations become highly non linear within a reducing interval, and the trade volume in the market collapses. The population now is composed by a stable trend chasers majority and, on the other hand, a stable fundamentalist minority, thus a stable higher homogeneity level which inhibits the switching mechanism and frustrates market volatility.

Then, from 0.11 on, price quickly converges to fundamental where the trend strategy is the most profitable, therefore the information exchange is biased by the profit channel disabling the switching, and stability in expectations is reached with trend chasers

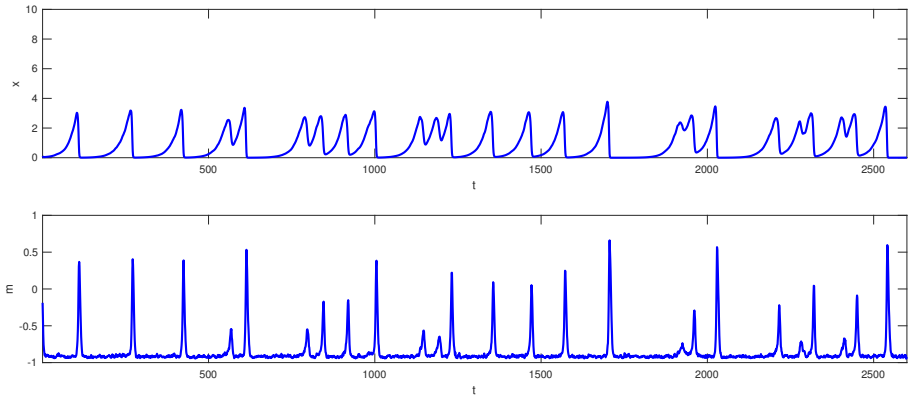


(a) 1% level of stubborn agents

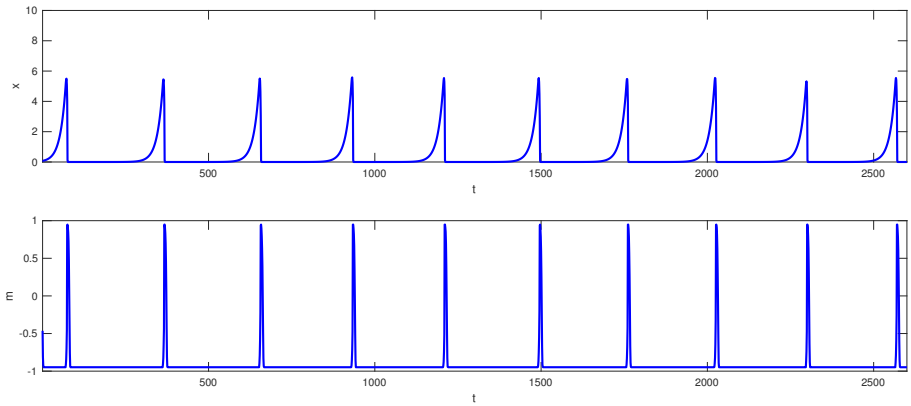


(b) 5% level of stubborn agents

Figure 3.2: Price and fractions dynamics with increasing level of stubborn agents



(a) $\beta = 3$



(b) $\beta = 100$

Figure 3.3: Price and fractions dynamics with increasing level of intensity of choice β

dominating despite the consistent presence of fundamentalist allowing local heterogeneity. Oscillations disappear.

Increasing the intensity of choice, again, produces regular and wider price fluctuations, long and constant period of trend chasers majority followed by short fundamentalists time frame, and a significant trading volume. The effects described above are cancelled out when the percentage of stubborn agent approaches 0.17, there, in fact, price converges to the fundamental value where chartist expectations dominance is neutralized by a sufficient threshold of fundamental agents ensuring stability in the market.

3.2 Random Graph

This network structure is built following the Watts & Strogatz model (1998). The starting point is a regular lattice with a degree $K = 25$, i.e. the number of links of each node, common to each individual. Then, the outcome is interpolated with the Erdos-Renyi graph where, with probability $\pi = 1$, each node suffers a rewiring process. The resulting structure has low average path lengths, low clustering coefficient and a degree distribution following a Poisson distribution. Although the same zero homophily level as for the fully connected scenario, frictions in the structure might be responsible for a slower information diffusion.

From 1% to 14% Stubborn Agents

Fast fluctuations in prices and expectations emerge, introducing percentages of stubborn agents, both fundamentalists and chartists. The price goes up pushed by over investing of trend chasers which dominate the market, then, because of negative excess return, they suffer losses while fundamentalists receives positive profits becoming the majority until the price reaches again the fundamental value. There, after few periods, chartists lead the market due to positive cost for the rational strategy, and a new bubble is triggered. It should be stressed how relatively slow is the bubble creation compare to its burst, which suddenly occurs.

An increase in the fraction of stubborn agents, until the threshold of 0.14 included, affects price fluctuations and the number of shares traded. Price dynamics becomes irregular while the amplitude of its oscillations shrinks from eight to less than four. The shares traded decrease, while expectations swings continue to be wild.

The price dynamics behaviour triggered by increasing the intensity of choice from 1 to 100 is dual: at first it forces price non linear fluctuations, while further rises make oscillations regular, less frequent, and wider in amplitude. Fractions fluctuations are wild and regular, swinging from long and constant trend chaser majority to short fundamentalist ruling periods during and after the bubble. In the same spirit, the trade volume starts again and become fleshy. Overall, the cycle dynamics is faster, because the burst happen now almost instantaneously.

Over 15% Stubborn Agents

Further rise of opinion leaders, until 0.21 included, strongly supports the former changes in price and shares traded: oscillations become highly non linear within a reducing interval, and the trade volume in the market collapses. The population now is composed by a stable trend chasers majority and, on the other hand, a stable fundamentalist minority, thus a stable higher homogeneity level which inhibits the switching mechanism and frustrates market volatility.

Then, from 0.22 on, price quickly converges to fundamental where the trend strategy is the most profitable, therefore the information exchange is biased by the profit channel disabling the switching, and stability in expectations is reached with trend chasers dominating despite the consistent presence of fundamentalist allowing local heterogeneity. Oscillations disappear.

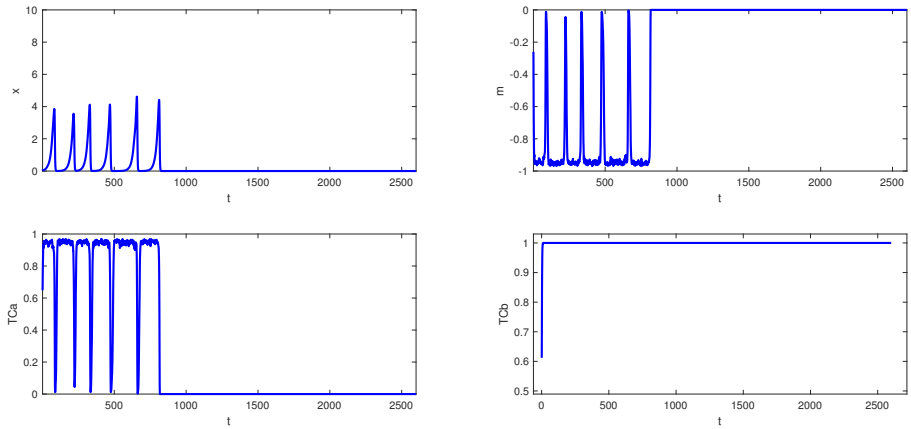
Increasing the intensity of choice, again, produces regular and wider price fluctuations, long and constant period of trend chasers majority followed by short fundamentalists time frame, and a significant trading volume. The effects described above are cancelled out when the percentage of stubborn agent approaches 0.34, there, in fact, price converges to the fundamental value where chartist expectations dominance is neutralized by a sufficient threshold of fundamental agents ensuring stability in the market.

3.3 Block Model

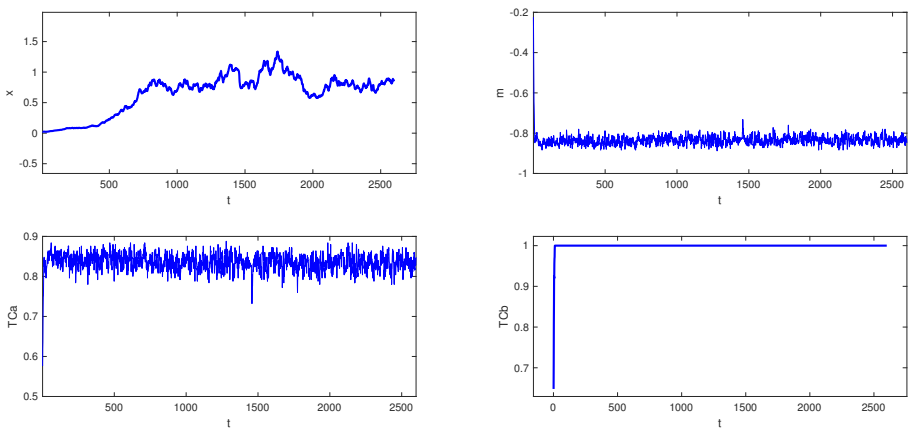
The maximum homophily level, regarding individuals' characteristics, is reached in a block model with two giant components, A, B, totally isolated from each other. Blocks are composed by fully connected network where heterogeneous agents with respect to expectations interact, while they are homogeneous regarding their characteristic. Thus, block A has eventually only stubborn fundamentalists and block B only stubborn chartists, while both having herding agents with both beliefs. One can think at the network adjacency matrix $A_{bm}(2 \times 2)$ whose elements are A_{fc} on the main diagonal and zero matrix on the counter diagonal.

From 1% to 10% Stubborn Agents

After the introduction of small percentage of stubborn agents, fundamentalists in block A and chartists in B, trend chasers quickly dominate the second block, while in block A expectations switching occurs until fundamentalists drive out from the network all trend chasers. Thus, fluctuations in prices emerge due to fractions oscillations in A, and their persistence relates on how fast rational agents win in the first block. The price goes up pushed by over investing of trend chasers which dominate both groups, then, because of negative excess return, they suffer losses while fundamentalists receives positive profits becoming the majority until the price reaches again the fundamental value. There, after few periods, chartists lead the market due to positive cost for the rational strategy, and a new bubble is triggered. Then, once only one expectation type, fundamental, leads block A, price converges to the fundamental value and the market becomes definitively stable.

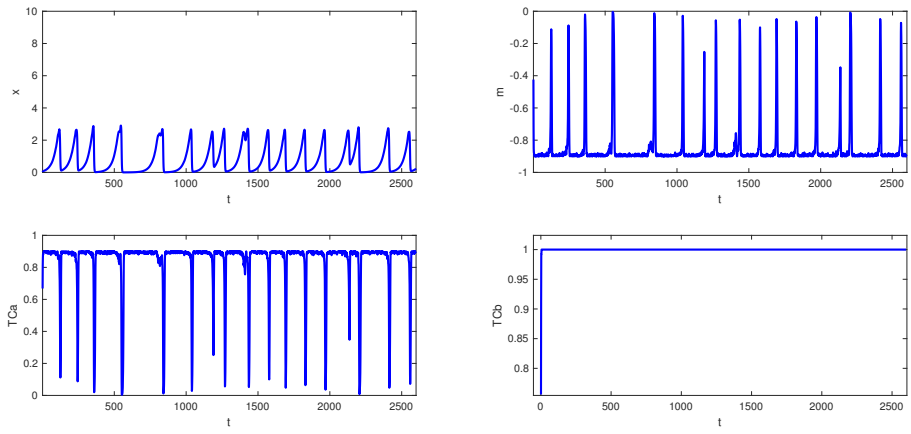


(a) 1% level of stubborn agents

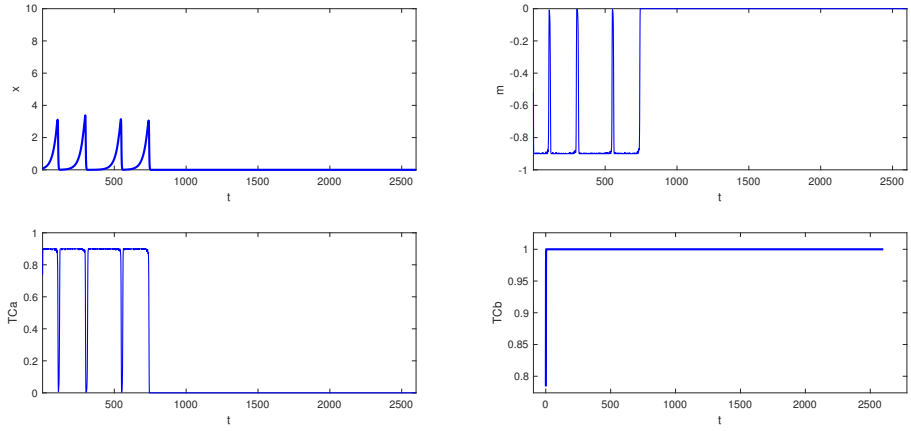


(b) 10% level of stubborn agents

Figure 3.4: Price dynamics and fractions evolutions in block A (bottom left) and block B (bottom right) with increasing level of stubborn agents.



(a) $\beta = 3$



(b) $\beta = 100$

Figure 3.5: Price dynamics and fractions evolutions in block A (bottom left) and block B (bottom right) with increasing level of intensity of choice β

An increase in the fraction of stubborn agents, until the threshold of 0.10 included, affects fraction fluctuations and, thus, market dynamics. Trend chasers presence in block A is not temporary any more, although the raised number of biased fundamentalist in that group, leading to persistent non linear price bubbles with decreasing amplitude, upper bound from six to less than two. See figure 3.4.

A differentiated fractions dynamics behaviour, triggered by increasing the intensity of choice from 1 to 100, arises in the two blocks: A is characterized by a quicker and constant majority of irrational traders before price bubble formation, and by a faster fundamental supremacy; in B trend chasers become leaders instantaneously. This features set off a price dynamics reduced in oscillations numbers, but regular and wider in amplitude. See figure 3.5.

Over 10% Stubborn Agents

Further rise of opinion leaders makes the price converging to fundamental where the trend strategy is the most profitable, therefore the information exchange is biased by the profit

channel disabling the switching, and stability in expectations is reached with trend chasers dominating block B and leading block A despite the consistent presence of fundamentalist allowing local heterogeneity. Oscillations disappear.

Increasing the intensity of choice, again, produces few regular, fast and wide price fluctuations where block A is characterized by long and constant period of trend chasers majority followed by short fundamentalists time frame, and a significant trading volume, while in B homogeneity is reached immediately. Then, if expectations heterogeneity is still present in A, a new boom rises. The effects described above are cancelled out when the percentage of stubborn agent approaches 0.17, there, in fact, chartist expectations dominate both groups but are neutralized by a sufficient threshold of fundamental agents in block A, ensuring stability in the market with price converging to the fundamental value.

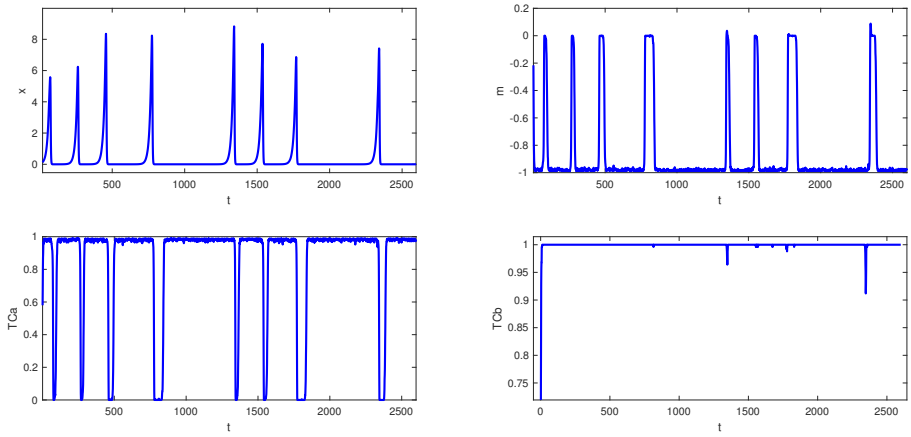
3.4 Barbell Network

Finally, adding several links between the two components of the block model, enables the study of intermediate homophily levels. In particular, two vectors of equal length, l, whose entries are randomly selected from the agents of the two components, A and B, are generated. Then, edges between every two elements is created, and the new adjacency matrix A_{bn} is considered as the input of the new network structure. The number of entries of the two vectors is increasing in order to obtain decreasing homophily level: if $l = 0$ we get the extreme block model, whether if $l > 0$ the network structure starts to resemble a barbell shape where increasing inter-groups communication is allowed.

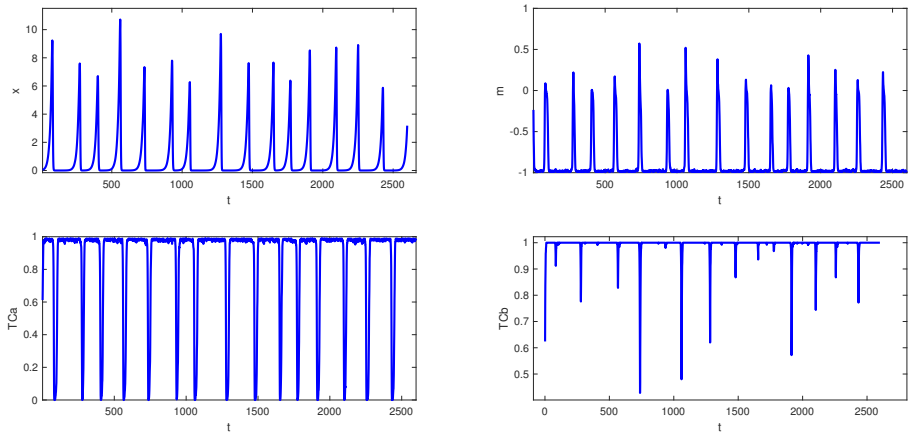
From 1% to 5% Stubborn Agents

After the introduction of small percentage of stubborn agents, fundamentalists in block A and chartists in B, trend chasers quickly dominate the second block with just few agents being affected by the possibility of inter communication and no contagion effect registered, while in block A extreme expectations switching occur. Thus, fluctuations in prices emerge due to fractions oscillations in A. The price goes up pushed by over investing of trend chasers which dominate both groups, then, because of negative excess return, they suffer losses while fundamentalists receives positive profits becoming the only expectation type present in block A until the price reaches again the fundamental value. There, after few periods, chartists come back in A via inter communication and lead the market, again, due to positive cost for the rational strategy, and a new bubble is then triggered.

Market dynamics are intensified allowing a growing inter communication between the two blocks. Component A experiments again extreme fractions fluctuations again, but dramatically frequent. Component B is now characterized by higher expectations volatility with fundamentalists suddenly being majority while vanishing slowly. We observe price fluctuations with equal amplitude, but higher in number and faster during busts.



(a) 10% new links



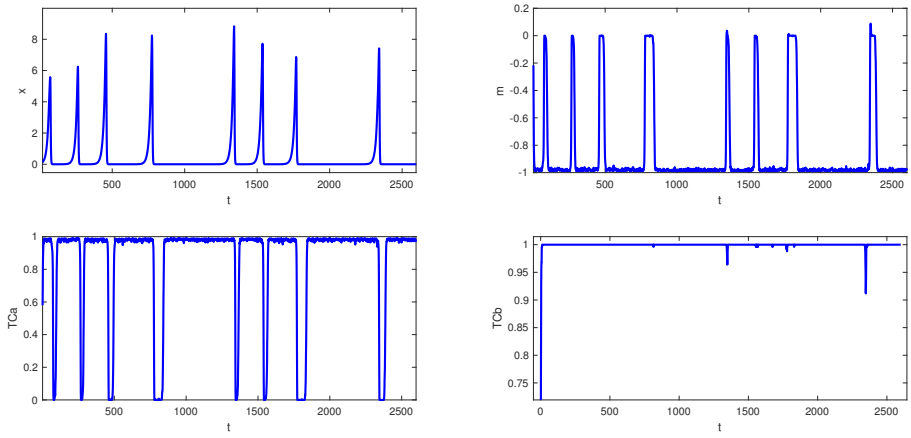
(b) 100% new links

Figure 3.6: Price dynamics and fractions evolutions in block A (bottom left) and block B (bottom right) with increasing inter communication.

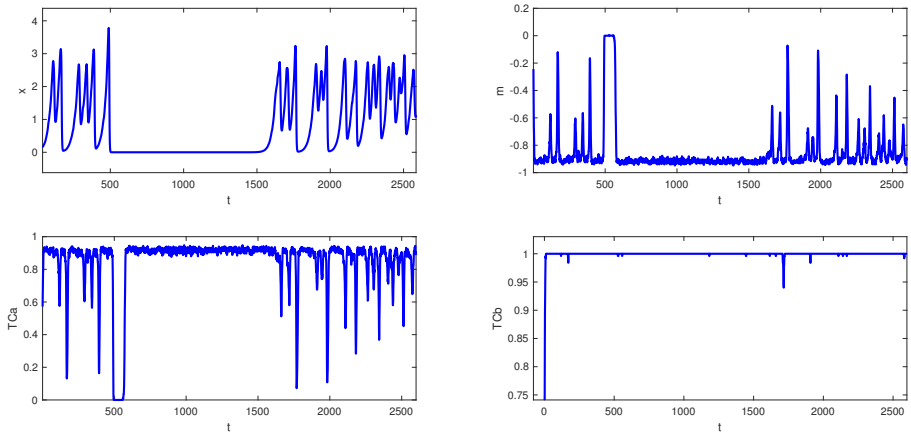
See figure 3.6.

An increase in the fraction of stubborn agents, until the threshold of 0.05 included, affects fraction fluctuations and, thus, market dynamics. Although fluctuations number and non linearity grows, they are no extreme any more and trend chaser always represent market majority. In particular irrational expectation is never driven out of block A and just irrelevant shocks are observed in B. Price oscillations become larger in number while their amplitudes shrink from ten to less than 4. See figure 3.7.

With low number of bridges ($l = 10\%$), increasing β from 1 to 100, at first, increases the number of fluctuations, extends the amplitude, let emerge fractions oscillations also in B. Then, increasing further β ($\beta = 100$), fluctuations become regular and less in number, faster during busts, although always wider, and extreme swinging of expectations in B now occurs and after fundamentalists lead this group, their presence in the market last longer. With large number of bridges ($l = 100\%$), although we already observe

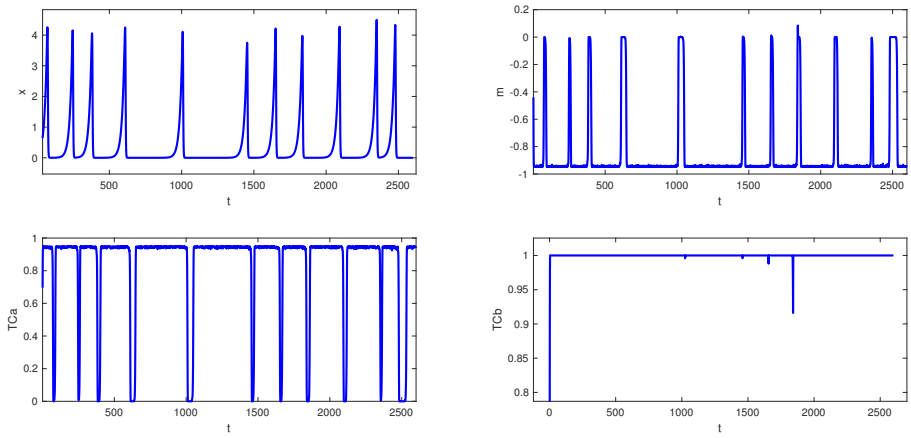


(a) 1% level of stubborn agents

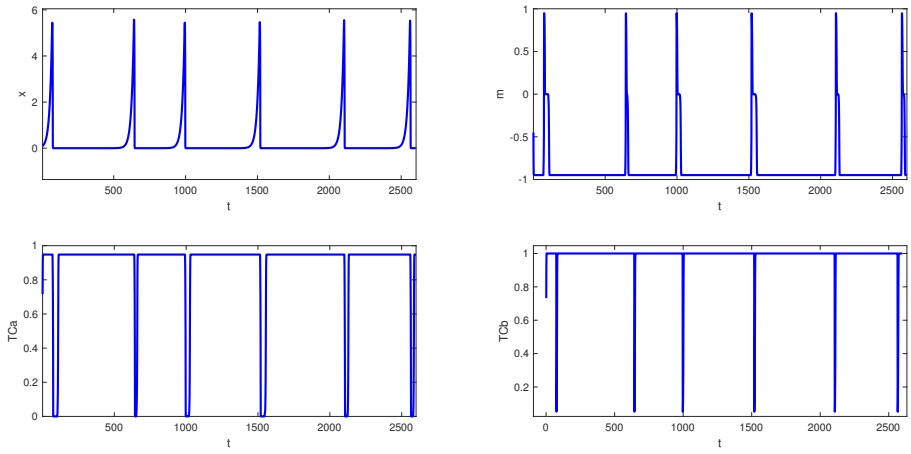


(b) 5% level of stubborn agents

Figure 3.7: Price dynamics and fractions evolutions in block A (bottom left) and block B (bottom right) with increasing level of stubborn agents while the percentage of new links is fixed.



(a) $\beta = 3$



(b) $\beta = 100$

Figure 3.8: Price dynamics and fractions evolutions in block A (bottom left) and block B (bottom right) with increasing level of intensity of choice β and fixed percentage of new links.

numerous fluctuations and fraction oscillation within block B, increasing β makes them wider in amplitude and larger in frequency at first, then it leads to more regular, less in number, faster during busts and extreme swinging of expectations in both blocks with sticky vanishing fundamentalists in block B. See figure 3.8.

Over 5% Stubborn Agents

Further rise of opinion leaders, until the threshold of 10% included, leads to a persistent although non regular majority of irrational agents in block A, and to their dominance with irrelevant shock in block B. Given the new expectations dynamics, price shows a non linear behaviour around a a very small non fundamental value. Further rise, makes the price converging to fundamental where the trend strategy is the most profitable, therefore the information exchange is biased by the profit channel disabling the switching, and stability in expectations is reached with trend chasers dominating block B and leading block A despite the consistent presence of fundamentalist allowing local heterogeneity. Price oscillations disappear, while fractions oscillations continue in block A although in a very small interval that always represent a consistent majority of trend chasers.

The effect of growing inter communication is vanishing with the increase of stubborn agents. Market dynamics is not affected by allowing a growing inter communication between the two blocks. A shows always a persistent although highly non linear majority of chartists, while block B is hit by very small fundamentalist shocks.

Increasing β from 1 to 100 wipes out the effect of a large number of opinion leaders: increases the number of fluctuations, wider amplitude and the interval is not any more around a non fundamental value, few fractions oscillations also in B again. Then, increasing further β ($\beta = 100$), fluctuations become regular and less in number, faster during busts, although always wider, and extreme swinging of expectations now occurs only in $A$with few shocks in B. The effects described above are cancelled out when the percentage of stubborn agent approaches 0.17, there, in fact, chartist expectations dominate constantly both groups but are neutralized by a sufficient constant threshold of fundamental agents in block A, ensuring stability in the market with price converging to the fundamental value.

4 Conclusions

We can conclude that a counter intuitive result emerges from the numerical analysis. A strong homophily should lead to expectations clusters and, thus, to persistent price fluctuations which, instead, occur only with intermediate homophily rate. Indeed, rising the homophily level to its maximum unity value, produces a more stable environment because of the different stable expectations dynamics characterizing the two blocks. On the other hand, in the barbell shape scenario, allowing inter communication and diminishing intra communication, hence reducing homophily, forces extreme expectations oscillations and,

thus, stronger and larger in number, price fluctuations.

An other important conclusion can be associated with the pure network structure, thus leaving out exogenous homophily considerations. In fact, a difference between fully connected and random graph which have the same level of homophily, occurs: frictions in communication slow the expectations diffusion process, and postpone all the dynamics effects characterized before.

Both previous results are related to the interpretation of stubborn agents role and intensity of choice effects. Increasing the percentage of stubborn agents at first guarantees local expectations heterogeneity, and therefore it makes the probability of switching increase via local interactions. Secondly, it leads to persistence majority of irrational traders as can be seen on the profit channel: sufficiently many stubborn fundamentalist prevents the price to take off and makes extrapolating the most profitable strategy.

Increasing the intensity of choice has a double effect too. Directly, in fact, higher intensity of choice brings more instability making agents more sensitive to changes in performances and with higher willingness to switch. The indirect effect, instead, appears through the social influence channel: higher β leads to lower heterogeneity in the population because everyone switches to the most profitable strategy, inducing, thus, lower probability of meeting different expectations and, therefore, slower switching.

References

[1] Daron Acemoglu, Asuman Ozdaglar, and Ali ParandehGheibi. "Spread of (mis) information in social networks". In: *Games and Economic Behavior* 70.2 (2010), pp. 194–227.

[2] Daron Acemoğlu et al. "Opinion fluctuations and disagreement in social networks". In: *Mathematics of Operations Research* 38.1 (2013), pp. 1–27.

[3] Torsten Arnswald. "Investment behaviour of german equity fund managers-an exploratory analysis of survey data". In: (2001).

[4] Robert Axelrod. "The dissemination of culture: A model with local convergence and global polarization". In: *Journal of conflict resolution* 41.2 (1997), pp. 203–226.

[5] William A Brock and Cars H Hommes. "A rational route to randomness". In: *Econometrica: Journal of the Econometric Society* (1997), pp. 1059–1095.

[6] William A Brock and Cars H Hommes. "Heterogeneous beliefs and routes to chaos in a simple asset pricing model". In: *Journal of Economic dynamics and Control* 22.8-9 (1998), pp. 1235–1274.

[7] Lauren Cohen, Andrea Frazzini, and Christopher Malloy. "The small world of investing: Board connections and mutual fund returns". In: *Journal of Political Economy* 116.5 (2008), pp. 951–979.

[8] Peter M DeMarzo, Dimitri Vayanos, and Jeffrey Zwiebel. "Persuasion bias, social influence, and unidimensional opinions". In: *The Quarterly Journal of Economics* 118.3 (2003), pp. 909–968.

[9] Benjamin Golub and Matthew O Jackson. "Naive learning in social networks and the wisdom of crowds". In: *American Economic Journal: Microeconomics* 2.1 (2010), pp. 112–49.

[10] Harrison Hong, Jeffrey D Kubik, and Jeremy C Stein. "Thy neighbor's portfolio: Word-of-mouth effects in the holdings and trades of money managers". In: *The Journal of Finance* 60.6 (2005), pp. 2801–2824.

[11] Ulrich Krause. "A discrete nonlinear and non-autonomous model of consensus formation". In: *Communications in difference equations* (2000), pp. 227–236.

[12] Lin Li et al. "Consensus, polarization and clustering of opinions in social networks". In: *IEEE Journal on Selected Areas in Communications* 31.6 (2013), pp. 1072–1083.

[13] Valentyn Panchenko, Sergiy Gerasymchuk, and Oleg V Pavlov. "Asset price dynamics with heterogeneous beliefs and local network interactions". In: *Journal of Economic Dynamics and Control* 37.12 (2013), pp. 2623–2642.